DOCUMENT RESUME

ED 478 192 SE 068 133

AUTHOR	Dudar, Abdur-Rahim Dib
TITLE	Logic for the English Teacher and Student.
PUB DATE	2002-02-00
NOTE	22p.; In: An Imperfect World: Resonance from the Nation's Violence. 2002 Monograph Series, Proceedings of the Annual Meeting of the National Association of African American Studies, the National Association of Hispanic and Latino Studies, the National Association of Native American Studies, and the International Association of Asian Studies (Houston, TX, February 11-16, 2002).
PUB TYPE	Reports - Descriptive (141) -- Speeches/Meeting Papers (150)
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	Higher Education; Logical Thinking; Mathematical Concepts; *Mathematical Logic; *Mathematics Instruction; Mathematics Skills; Probability; Set Theory
IDENTIFIERS	Boolean Algebra

ABSTRACT

This paper suggests that logic consists of a collection of propositions and operations of negation, conjunction, disjunction, implication, and equivalence. It points out that the operations on dispositions depend upon the truth-value of the propositions involved. This raises the questions, How do we know whether a proposition is true or false? and How do we construct a proposition? (SOE)

Reproductions supplied by EDRS are the best that can be made

ED 478 192

LOGIC FOR THE ENGLISH TEACHER AND STUDENT

ABDUR-RAHIM DIB DUDAR, ED.D.

BENEDICT COLLEGE

COLUMBIA, SOUTH CAROLINA

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

1

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

- This document has been reproduced as received from the person or organization originating it.
- Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

SE068133

BEST COPY AVAILABLE

Logic for the English Teacher and Student

Introduction

Sound decisions and great accomplishments are based on logical thinking and not emotional appeal. Logic develops in the individual a sense of scrutiny, which protects him/her against the fallacies of reasoning, fallacies which might arise in commercial advertisements and political propaganda. Logic is an excellent guide to truth; without it, we are apt to fail. Logic is not only important in our daily life. It is the tool every branch of knowledge uses to become comprehensible. The scientific laws, for instance, are logical conclusions based upon study of patterns, resulting from observations and means of experimentation.

Logic consists of a collection of propositions and operations of negation, conjunction, disjunction, implication, and equivalence. The operations on propositions depend upon the truth value of the proposition(s) involved. The question one may raise is this: How do we know whether a proposition is true or false? And for that matter, how do we construct a proposition? In the first place, what is a proposition? If we say a proposition is a

meaningful sentence, then what is a meaningful sentence which can be called true or false?

In the late sixties I happened to have come in contact with a book written by Morris Cohen and Ernest Nagel, Introduction to LOGIC and SCIENTIFIC METHOD (1934). The authors introduce the concepts of extension and intension of a name. I elaborated on these ideas and extended them to a construction of a logic system, a Boolean Algebra.

Extension and Intension of a Term.

A **Term** is the name by which a thing or some aspect of a thing is symbolized. The word "thing" is used in its wide sense to include living and non-living things, qualities, quantities, ideas and actions. The following are terms:

> Teacher, Pennsylvania, peace, herd, tiger, beauty, running, swimming, wise man,
>
> The White House, four-sided-plane figure

An **Extension** of a term, written **Ext(Term)**, is the class of all objects to which the term applies past, present and future. When a term denotes nothing its extension is said to be **empty**.

The extension of the term "teacher," **Ext(Teacher)** is the class of all persons whose profession is teaching, past, present and future. And the extension of the term four-sided-plane figure,

> **Ext(Four-sided plane figure)** is the class of all geometric figures in a plane that are closed and have only four sides.

The extension of the U.S. King 1964 is an empty class of objects.

When the term "square" is mentioned something is usually conceived about its characteristics. For example, a square is a plane figure; its sides are equal; it has four right angles; it has four vertices; it has four corners; its area is the length of the side squared; its perimeter is the product of four and the length of one of its sides. What is conceived about "square" is a class of attributes of the square itself. Such attributes will define what a square is and applies to each member of the class of all squares.

The **Intension** of a term, denoted **Int(Term)**, is the class of all attributes of the term itself.

Therefore, a term expresses its extension, the class of all objects to which it applies, and its intension, the class of all attributes of the objects to which it applies. Given a certain term, there is a set of attributes common to all objects in its extension. Such a set of attributes is said to be **essential** when the attributes

define the object to which the term applies no single attribute may be excluded and none more may be included without changing the definition of the object itself. Therefore, an essential set of attributes constitutes the definition of a term precisely. For a parallelogram, the essential set of attributes would be (a) It is a quadrilateral, and (b) Its opposite sides are parallel.

Simply stated the extension and intension of a term are **sets**. Although the concept of a set is undefined, a given set should be fully recognizable. We must have well-defined sets so that we can identify whether an object belongs or does not belong to the set. In other words, a set S is said to be **well-defined** if and only **if** given any object p either p belongs to S or , exclusive or, p does not belong to S. Using the above definition, the **extension of a term** is well-defined whenever its set is well-defined. A similar definition may be given to a well-defined **intension of a term**. Thus, a **term** is said to be well-defined if and only if its extension and intension are well-defined.

Simple Propositions

Only well-defined terms will be acceptable in our study of logic. Otherwise a statement formed using ambiguous terms will

remain ambiguous and for that matter one cannot assign a truth value to such statements. To construct the simple propositions we need to rely on set theoretic concepts such as "Is a member of" ($\in$), "Is a subset of" ($\subseteq$), intersection of two sets ($\cap$), equality of sets (=), and the empty set (ϕ).

Consider the following statements:

(1) Isaac Newton discovered the Law of Inertia.

(2) Desiree loved Napoleon I.

(3) All sulfates are salts.

(4) Some shoe makers are poor.

(5) No Muslim is Christian.

(6) Some Americans do not support the president.

Using properties of sets, the statements may be written as follows:

(1)' Isaac Newton = The discoverer of the Law of Inertia.

(2)' Desiree $\in$ Ext(Female lovers of Napoleon I).

(3)' Ext(Sulfates) $\subseteq$ Ext(Salts).

(4)' [Ext(Muslim)] $\subseteq$ [Ext(Christian)]'

(5)' [Ext(shoe-maker)] $\cap$ [Ext(Poor person)] $\neq \phi$.

(6)' [Ext(Americans)] $\cap$ [Ext(President Supporter)]' $\neq \phi$.

In general, given any universe U, and x, and s members of the universe U, and X and Y subsets of the universe U, there are only six possibilities for statements about x, s, X, and Y with respect to U:

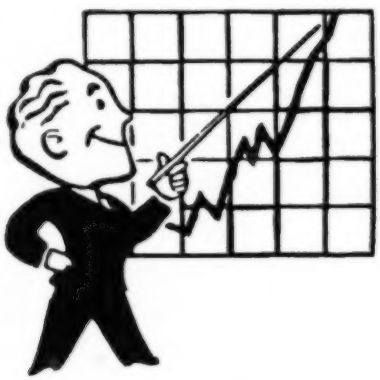
(1) $x = s$, (2) $x \in X$, (3) $X \subseteq Y$,
(4) $X \subseteq Y'$, (5) $X \cap Y \neq \phi$, and (6) $X \cap Y' \neq \phi$

These statements are called **variable statements on U**.

Notice that $x = x$, $s = s$, $X=X$, $Y=Y$ are true statements for any member or subset of U as = is reflexive. Therefore stating that $x \neq x$, $s \neq s$, $X \neq X$, $Y \neq Y$ are false statements regardless of what x, s, X and Y are so long as they are either elements of U or subsets of U. This means that for every true statement on the universe U, a false statement is associated with that true statement. If x and s are distinct members of U, then saying $x = s$ is a false statement regardless whatever x and s are so long as x and s are distinct members of U. On the other hand, stating that $x \neq s$ would be a true statement. This leads us to say that if a statement p is true then the statement not p is false, and if the statement not p is true, then the statement p is false.

Common sentences may be converted to set theoretic statements without making any changes in meaning.

The Babylonians wrote their laws.	The Babylonians are writers of their laws. **X** **Y** Thus, $\mathbf{X} \subseteq \mathbf{Y}$.
No democracy succeeds in under-developed countries.	Democracies are not successful in under- **X** developed countries **Y** Thus, $\mathbf{X} \subseteq \mathbf{Y}'$
Some Americans do not play football.	Some Americans are not football players **X** **Y** Thus, $\mathbf{X} \cap \mathbf{Y}' \neq \phi$
Some students work hard.	Some students are hardworking persons. **X** **Y** **Thus, $\mathbf{X} \cap \mathbf{Y} \neq \phi$**



Operations on Propositions

Suppose some **alien's universe**, U, consists of three objects { a, b, c }. The alien's simple propositions number **289** sentences! Notice that all these sentences, statements, are demonstrated to be

true or false using properties of set theory. Now the alien resorts to negation, conjunction, disjunction, implication, exclusive disjunction and equivalence to build his language of true or false statements. He would write new statements using these operations such as

$$\{[p\wedge(q\vee r)]\rightarrow[(s\vee t)\vee(v\wedge n)]\}\leftrightarrow\{[\sim(\sim y)\wedge u]\leftrightarrow w\wedge[(c\vee d)\wedge m]\}$$

where each letter represents some simple proposition. Each one of these combinations make up a new sentence or statement called a **proposition**. The question to raise is: How would this alien defines the **truth values** of propositions. This intelligent alien comes up with tables to define the truth value of each proposition. He writes:

p	T	F
~p	F	T

Negation

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

p	q	p∨q
T	T	T
T	F	T

F	T	T
F	F	F

Disjunction

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Implication

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Equivalence

p	q	p ⩒q
T	T	F
T	F	T
F	T	T
F	F	F

Exclusive Or

Now the alien can have an infinite number of propositions based on a boring universe of three elements! Every proposition has a truth value inherited from those truth values of simple propositions. **Similar construction may be done for any universe of any number of elements with similar truth value tables.**

Special Properties

There are few theorems relevant to our proper verbal communication whose statements and proofs are given below. One deals with double negation, " I don't know nothing," and the second deals with negation of compound sentences such as," It is false that Munir went to town and took Fatimah with him or Rasheed played his musical instrument and refused to go fishing." Theorem 1 states that the negation of the negation is confirmation. Theorem 2 has two parts. One that says the negation of a conjunction is a disjunction of the negations and the second states that the negation of a disjunction is the conjunction of the negations.

To prove a theorem in logic we merely use a truth table that consists of all possible constituents of the total proposition and assign the various values for the truth values of the basic proposition and compute the respective truth values of the various constituent propositions.

Theorem 1. ~ (~p) ↔ p

Proof.

p	~p	~ (~p)	~ (~p) ↔ p
T	F	T	t T t
F	T	F	f T f

"I don't know nothing" becomes "I know something." The speaker meant he/she does not know anything. Logically speaking, the speaker is negating what he/she is saying! The next theorem addresses the negation of a compound sentence.

Theorem 2. (DeMorgan's Laws)

(a) $\sim (p \wedge q) \leftrightarrow (\sim p) \vee (\sim q)$

(b) $\sim (p \vee q) \leftrightarrow (\sim p) \wedge (\sim q)$

Proof.

(a)

p	q	~p	~q	p∧q	~(p∧q)	(~p)∨(~q)	~ (p ∧ q) ↔ (~p) ∨ (~q)
T	T	F	F	T	F	F	f T f
T	F	F	T	F	T	T	t T t
F	T	T	F	F	T	T	t T t
F	F	T	T	F	T	T	t T t

(b)

p	q	~p	~q	p∨q	~(p∨q)	(~p)∧(~q)	~(p ∨ q) ↔ (~p) ∧ (~q)
T	T	F	F	T	F	F	f T f
T	F	F	T	T	F	F	f T f
F	T	T	F	T	F	F	f T f
F	F	T	T	F	T	T	t T t

The compound sentence,

> "It is false that Munir went to town and took Fatimah with him or Rasheed played his musical instrument and refused to go fishing"

may be rewritten as,

> "It is false: (Munir went to town and took Fatimah with him) or (Rasheed played his musical instrument and refused to go fishing.)"

DeMorgan's laws may be applied twice to get the final answer. DeMorgan's Law is applied first to the disjunction of the two sentences and get:

> It is false that (Munir went to town and took Fatimah with him) and It is false that (Rasheed played his musical instrument and refused to go fishing."

On the second application of DeMorgan's theorem, we will be negating two conjunctions and the final answer becomes:

> "Munir did not go to town or he did not take Fatimah with him and Raheed did not play his musical instrument or he did not refuse to go fishing."

The following third theorem plays a big role in our definitions and equivalences.

From Geometry, the definition of two parallel lines is given as, "Two lines are parallel if and only if they lie in the same plane and can never meet however extended," and the corresponding theorem "Two lines are parallel is equivalent to the alternate

interior angles on a transversal are congruent" remind us of the use of "if and only if; the necessary and sufficient condition(s) to; if-then and conversely; and, equivalence." The theorem below demonstrates that a definition and an equivalence are made up of two implications that hold jointly. Simply, the theorem states that x implies y and y implies x is the same as saying that x and y are equivalent. The term "**tautology**" is used to mean that the statement is always true regardless of the truth values of the constituent statements.

Theorem 3 $(x \leftrightarrow y) \Leftrightarrow [(x \rightarrow y) \wedge (y \rightarrow x)]$ is a tautology.

Proof.

x	y	$x \rightarrow y$	$y \rightarrow x$	$(x \rightarrow y) \wedge (y \rightarrow x)$	$x \leftrightarrow y$	$(x \leftrightarrow y) \Leftrightarrow [(x \rightarrow y) \wedge (y \rightarrow x)]$
T	T	T	T	t T t	T	t T t
T	F	F	T	f F t	F	f T f
F	T	T	F	t F f	F	f T f
F	F	T	T	t T t	T	t T t

The last column in the table shows that whatever the truth values of x and y, the proposition $(x \leftrightarrow y) \Leftrightarrow [(x \rightarrow y) \wedge (y \rightarrow x)]$ is true.

To be consistent with set theory, implication is analogous to "is a subset of." The proof of the theorem is left as an exercise. Notice that 0 represents a false proposition.

Theorem 4 (Consistency Theorem)

For any two propositions, the following propositions are tautologies:

(1) $(x \rightarrow y) \Leftrightarrow [(x \vee y) \leftrightarrow y]$ (2) $(x \rightarrow y) \Leftrightarrow [(x \wedge y) \leftrightarrow x]$ (3) $(x \rightarrow y) \Leftrightarrow [x \wedge (\sim y) \leftrightarrow 0]$

Arguments

Now we turn our attention to an application of logic. In any logical system, a true proposition is sometimes referred to as a **premise**. A series of premises, at least one of which is given to substantiate another, is called an **argument**. The proposition which is said to follow from the given premises is called the **conclusion** of the argument.

To prove that a conclusion follows from the premises one may place all premises in a compound proposition and verify whether the statement is true. A visual method to verify the validity of an argument is done using Euler-Venn diagrams. To disprove an argument one must provide a counterexample and not merely depend on a diagram.

Consider the following argument assuming all terms are well-defined and the statements (1) and (2) are premises.

(1) All doctors value the human life.

(2) People who value the human life enroll in a defensive driving course

---

Therefore (3) All doctors enroll in a defensive driving course. Using logical symbolism, let

P: People who are doctors

Q: People who value human life

R: People who enroll in a defensive driving course.

Then, the argument is stated as follows:

(1) $P \rightarrow Q$
(2) $Q \rightarrow R$

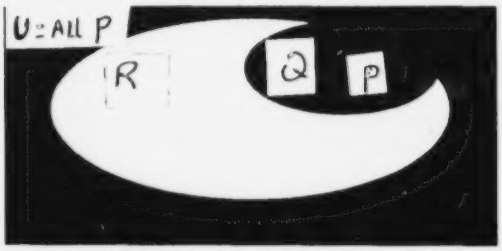
Therefore, $P \rightarrow R$.

The argument may be represented by a compound proposition whose validity may be verified by a truth table as we did previously in proving theorems. Thus one may prove: $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \Rightarrow (P \rightarrow R)$ is a tautology

Visually we may verify the validity of the conclusion using set theory.

$P \subseteq Q$
$Q \subseteq R$

Therefore, $P \subseteq R$



BEST COPY AVAILABLE

Notice that the conclusion is true with respect to the assumptions made. But these assumptions are themselves not true if one tries to verify their validity. For instance, not all doctors value the human life although they are supposed to. Even if all doctors value the human life, not everyone of them will take a defensive driving course. And it is not true that everyone who values the human life will enroll in a defensive driving course. In other words, one must evaluate the assumed premises as to their validity in order to make sound decisions. When one does not verify the validity of the assumptions while such assumptions are really false, the conclusion obtained is said to be **fallacious**. Therefore the fallacy in the above argument arises only from the validity of the premises in real life and not through an error in the conclusion as the following example shows.

Assume statements (1) and (2) are premises in the following argument:

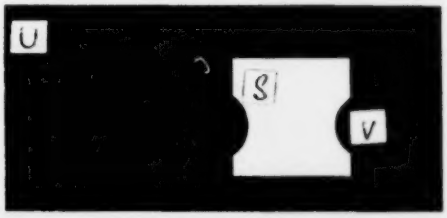
(1) Some salts are sulfates

(2) Some salts are chlorides

Therefore (3) Some chlorides are sulfates.

To demonstrate the fallacy of this argument, let us denote by U the set of all chemical compounds, S the set of all salts, T the

set of all sulfates, and V the set of all chlorides. Then premise (1) states that $S \cap T \neq \phi$ and (2) states that $S \cap V \neq \phi$. In a diagram,



Notice that $S \cap V = \phi$ rather than $S \cap V \neq \phi$ as (3) asserts.

Although this diagram demonstrates the falsehood of the conclusion, mathematically it is not good enough. Whenever a statement is to be shown false, one must provide a counterexample. A **counterexample** is one that satisfies all the premises and results in a different conclusion. For a counterexample consider the following situation to show the fallacy of the conclusion in the previous example.

Counterexample

Let $U=\{a, b, c\}$, $S=\{a, b\}$, $T=\{a\}$, and $V=\{b\}$. Then, $S \cap T=\{a\} \neq \phi$, and $S \cap V=\{b\} \neq \phi$. But $S \cap V = \phi$.

To **avoid fallacies**, we may follow the following steps:

(1) Check whether the terms are well-defined

(2) Assign the correct truth values to the given propositions

(3) Avoid mistakes in the reasoning process.

BEST COPY AVAILABLE

(4) In case an argument consists of complex propositions, split it into simple propositions and evaluate.

For example, suppose the following premises were given:

(1) Mary is a teacher, housewife or nurse.

(2) If Mary is not a teacher, she is not a housewife.

(3) Mary is not a teacher.

Show (4) Mary is a nurse and not a housewife.

Proof.

Let P: Mary is a teacher

Q: Mary is a housewife

R: Mary is a nurse.

Premise (1) becomes $P \vee Q \vee R$ is true; premise (2) becomes $(\sim P) \rightarrow (\sim Q)$ is true; and premise (3) becomes $\sim P$ is true. Since $\sim P$ is true and $(\sim P) \rightarrow (\sim Q)$ is true, it follows that $\sim Q$ is true; hence Q is false using the truth tables for implication and negation. So, Mary is not a housewife.

But premise (3) states that $\sim P$ is true. So P is false as Mary is not a teacher. Since $P \vee Q \vee R$ is true with P and Q being false, then R must be true. That is Mary is a nurse using the truth table for a disjunction.

For a compound statement, the argument may be written as

$$[(P \vee Q \vee R) \wedge (\{ (\sim P) \rightarrow (\sim Q)\} \wedge (\sim P))] \Rightarrow Q$$

and using a truth table verify that the proposition is true.

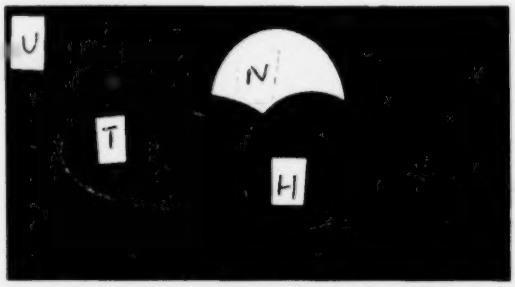
A Venn-Euler diagram may be used to demonstrate the results.

Let U=Set of all people

T=Teachers

H=Housewives

N=Nurses



Mary $\notin$ T and so, Mary $\notin$ H. Hence, Mary $\notin$ T$\cap$H, and Mary $\notin$ T$\cap$N, Mary $\notin$T$\cap$(H$\cap$N) and, Mary $\notin$H$\cap$N. Mary $\notin$ H, implies Mary $\in$H′. Therefore Mary $\in$ N exclusively and Mary $\in$H′.

In conclusion, in a society as complex as the United States, logic plays a major role in our daily life and our language. Please take note!

BEST COPY AVAILABLE

Bibliography

Allendoerfer, C.B. and Oakley, C. O. *Principles of Mathematics*. New York: McGraw-Hill Book Company, Inc. (1955).

Cohen, Morris and Ernest Nagel. *Introduction to LOGIC and SCIENTIFIC METHOD*. NY: Harcourt, Brace and Company (1934).

Stoll, Robert. *Set Theory and Logic*. San Francisco: W.H. Freeman and Company (1961).

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

ERIC REPRODUCTION RELEASE

I. Document Identification:

Title: An Imperfect World: Resonance from the Nation's Violence (2002 Monograph Series)

Editor: Lemuel Berry, Jr., Ph.D.

Corporate Source:

Publication Date: 2002

II. Reproduction Release:

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, Resources in Education (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please check one of the following three options and sign the release form.

_✓_ Level 1 - Permitting reproduction and dissemination in microfiche or other ERIC archival media (e.g. electronic) and paper copy.

____ Level 2A - Permitting reproduction and dissemination in microfiche and in electronic media for ERIC archival collection subscribers only.

____ Level 2B - Permitting reproduction and dissemination in microfiche only.

Documents will be processed as indicated provided reproduction quality permits. If permission to reproduce is granted, but no option is marked, documents will be processed at Level 1.

Sign Here: "I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries."

Signature: Lemuel Berry Jr

Position: Executive Director

Printed Name: Lemuel Berry, Jr.

Organization: NAAAS, NAHLS, NANAS, IAAS

Address: PO Box 325
Biddeford, ME 04005-0325

Telephone No: 207-~~282-1925~~ 839-8009

Date: 5/20/03

III. DOCUMENT AVAILABILITY INFORMATION (FROM NON-ERIC SOURCE):

If permission to reproduce is not granted to ERIC, or, if you wish ERIC to cite the availability of the document from another source, please provide the following information regarding the availability of the document. (ERIC will not announce a document unless it is publicly available, and a dependable source can be specified. Contributors should also be aware that ERIC selection criteria are significantly more stringent for documents that cannot be made available through EDRS.)

Publisher/Distributor:
Address:
Price:

IV. REFERRAL OF ERIC TO COPYRIGHT/REPRODUCTION RIGHTS HOLDER:

If the right to grant this reproduction release is held by someone other than the addressee, please provide the appropriate name and address:

Name:
Address:

V. WHERE TO SEND THIS FORM:

Send this form to the following ERIC Clearinghouse:

However, if solicited by the ERIC Facility, or if making an unsolicited contribution to ERIC, return this form (and the document being contributed) to:

ERIC Processing and Reference Facility
4483-A Forbes Boulevard
Lanham, Maryland 20706

Telephone: 301-552-4200
Toll Free: 800-799-3742
FAX: 301-552-4700
e-mail: info@ericfac.piccard.csc.com
WWW: http://ericfacility.org